Modeling of Solid/Liquid Expression for Cellular Materials

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A general theory is developed for the mechanical expression of agrofood, cellular materials. The so-called, Liquid-containing biporous particles expression model considers liquid transfer within a network of three different volumes: extraparticle, extracellular and intracellular volumes. The system of partial differential equations is solved for the expression under constant pressure, allowing calculation of the total layer settlement, as well as the deformation of extraparticle, extracellular and intracellular volumes. The model is able to predict the behavior of four different steps in the consolidation stage: the primary deformation and the creep deformation of extraparticle volume, and the deformation and deliquoring of both extracellular and intracellular volumes. The model is applied to the hydraulic pressing of rapeseeds. The theoretical model agrees well with experimental data for the overall range of pressing time.

Introduction

Mechanical expression of cellular materials, as it is largely used in various agrofood processes, is an important unit operation in the food industry: extraction of fruit juices, extraction of vegetable oils in oilseed crushing, dewatering of fibrous materials (sugar beet pulp, fooder plants), dewatering of waste water sludges, and so forth. Currently, most of the articles, which tend to describe the modeling of mechanical expression of cellular materials, are just an adaptation of the modeling of mineral cake expression; consequently, they are rather inappropriate.

In fact, numerous authors have used the filtration/consolidation theory for soils (Terzaghi, 1925). Shirato et al. (1970, 1971) and Körmendy (1974) have applied it for varying pressures and rates in the mechanical expression of mineral cakes, while Neuper and Staudinger (1984), Murase et al. (1987, 1988), and Vorobyov et al. (1993) have developed it with varying expression coefficients for highly compressed cakes. Tiller and Horng (1983) have introduced the theory of hydraulic expression for compressible filter cakes. Also, some authors developed the creep consolidation theory for the expression of clay cakes. This is conceptually close to Terzaghi's theory, but it accounts for two periods of consolidation: the primary and the creep (or secondary) consolidations (Gibson and Lo, 1961; Florin, 1959). Shirato et al. (1974, 1978, 1986), and Rebouillat et al. (1985) have used creep consolidation models for both constant pressure and rate, and varying pressure and rate.

The mechanisms for expression of mineral cakes and cellular materials are basically different. With mineral cakes, the cake is initially saturated by the liquid, and cake deformation depends only on the reorientation of individual, incompressible particles within the cake. With biological and cellular materials, the expression mechanism is much more complex. The liquid is stored mainly in cells; some gas may also be present in the cellular structure. During expression the air dissipation and the cell rupture significantly modify the cake compressibility. Also, intraparticle and extraparticle volumes are compressible. Thus, the reorientation of the liquid-containing particles and the evolution of the particle microstructure affect the local stress gradients, and consequently the expression performances. Although the mechanical expression of biological and cellular materials obeys a very complex mechanism, the authors have used the aforementioned theory and models, under constant pressure (Austmeyer, 1987; Sivala et al., 1991; Singh and Singh, 1991; Zaiats et al., 1993; Buttersack and Buchholz, 1993), or under linear increase of

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load (Mrema and McNulty, 1985). Schwartzberg (1978, 1983) and Schwartzberg et al. (1979) modified the Shirato's model by introducing the specific bulk volume per unit of dry nonexpressible solids, instead of a void ratio. Thus, the authors have studied the constant rate expression and the relaxation properties of various food cellular materials. Based on the same concept, Rebouillat et al. (1993) proposed a simplified model for the constant rate expression.

Recently, Lanoisellé et al. (1994) considered a new original mechanism, assuming that cellular materials consist of two porous systems, or volumes: intraparticle and extraparticle volumes. Thus, the authors developed a mathematical model for the expression of cellular materials that takes into account liquid flows from the intraparticle volume. Through a very simple version of the model, the authors could mesh satisfactorily with experimental data (hydraulic pressing of rapeseeds) only for low-pressure experiments (below 30 bar), for relatively short expression times (less than 20 min).

This article presents a general theory for the mechanical expression of biological and cellular materials. It is based on the Liquid-Containing Biporous Particles Expression Model. In fact, during expression of cellular materials, a large quantity of liquid is stored inside the cells and the extracellular volume. As the cake is deformed, the liquid flows from the interior of the cells to the extracellular volume, and then from the extracellular volume to extraparticle volume (free space between the particles). Expression performances depend on the flow properties of the liquid between these different volumes, as cake properties change significantly from the initial to the final stage of mechanical expression. The Liquid-Containing Biporous Particles Expression Model is an abstractive representation of the presscake, and the present theory aims at describing the liquid flow within such a conceptual cake, as well as the cake properties. Then the theoretical behavior of mechanical expression is compared with experimental observations.

Presentation of the Liquid-Containing Biporous Particles Expression Model

When starting a hydraulic pressing of agrofood materials, the cake initially consists of a fixed bed of finite elementary particles. This defines two different volumes: the intraparticle volume, which is the volume occupied by all particles in the bed, and the extraparticle volume (or interstitial volume), which represents the free space between particles; the extraparticle volume is distributed as a random network of channels (Figure 1a). The intraparticle volume usually contains the liquid to be expressed, while the extraparticle volume is occupied mostly by air. A small amount of liquid may appear around particles, particularly when some pretreatment processes are applied earlier to facilitate mechanical expression. The volume of the cake is of course much larger than the volume of the liquid-containing particle. Thus, the characteristic length of the cake, h (the height of the cake), is much larger than the characteristic length of the elementary particle, L. Any elementary particle consists of a well-organized network of cells that communicate by several plasmodesmata. Structural characteristics and the properties of the cell network depend on the cellular material. Thus, the intraparticle volume may be defined as an organized network of finite volumes: the intracellular volume, which is the volume occupied by all cells in the elementary particle, and the extracellular volume, which represents the noncell volume (Figure 1b). The majority of the liquid is trapped in the intracellular volume. The volume of the elementary particle is much greater than the volume of an individual cell. In which case, the characteristic length, L, is much greater than the characteristic length of the cell, d.

As shown by Lanoisellé et al. (1994), monodirectional hydraulic pressing proceeds as three successive stages. At the beginning of the first stage, the pressure applied by the compressive piston, P_E, is divided in two parts: one part is transferred to the particles, and is called the *effective pressure*, P_{S1}; while the other part, P_1, allows the compression and evacuation of the air from the cake through the extraparticle channels. As P_E increases, P_{S1} increases, causing the initial reorientation of the elementary particles, and consequently the initial decrease of the extraparticle volume. Liquid pressures, P_2 and P_3, relative to extracellular and intracellular volumes, respectively, also increase, allowing the liquid to emanate from the intraparticle volume to the extraparticle volume. The first stage ends when the first drop of liquid flows out of the cake. During the second stage, the extraparticle air is progressively replaced by liquid, and an air/liquid mixture is evacuated from the cake. The instantaneous flow rate of liquid increases rapidly up to a maximum that determines the end of the stage (Figure 2). Then the air is totally eliminated. The third stage of hydraulic pressing starts at the maximum

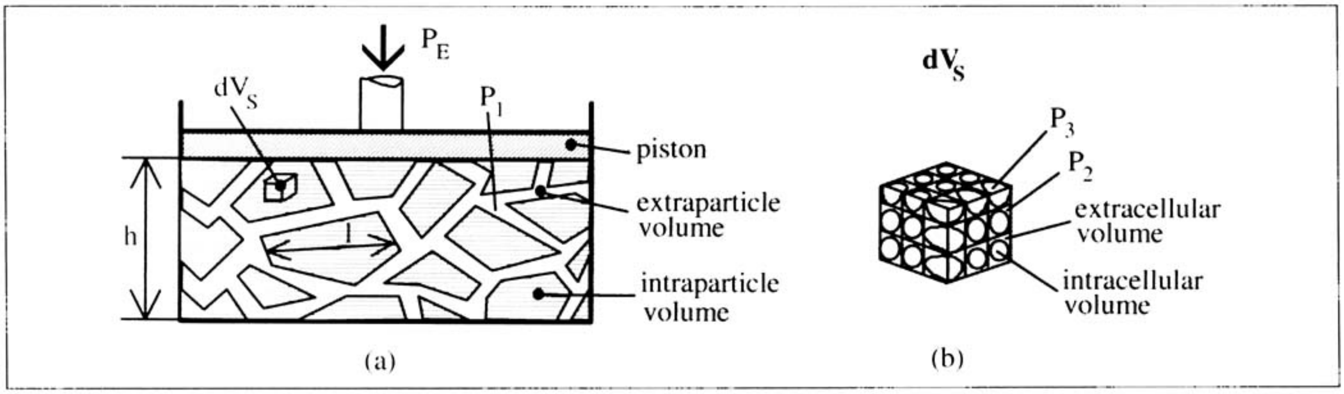

Figure 1. Liquid-containing biporous particles expression model.

(a) Extraparticle and intraparticle volumes. (b) Elementary volume of cellular particles.

instantaneous flow rate of liquid, when the intracellular, extracellular and extraparticle volumes are completely filled with liquid. This is the consolidation stage of the presscake. Let t_0 be the initial time of the third stage; then the initial liquid pressure in the extraparticle volume $P_1(z, t_0) = P_E - P_{S1}(z, t_0)$. Expression models for homogeneous semisolid materials (Shirato et al., 1986) assume that the applied pressure P_E is transferred initially only to the liquid phase ($P_E = P_1(z, t_0) = P_{S1}(z, t_0) = 0$). For the expression of cellular materials the initial liquid can be expressed from the intracellular and extracellular volumes (during the first and second stages of the process), only if $P_{S1}(z, t_0) \neq 0$. As the local solid pressure P_{S1} increases during the third stage of expression, the initial value of liquid pressure $P_1(z, t_0)$ is maximal. One can also assume for the initial conditions of the third expression stage that $P_1(z, t_0) = P_2(z, t_0) = P_3(z, t_0)$, which corresponds with the standard assumption of the filtration/consolidation theory about the equality of liquid pressures in all pores across the cake section. According to these assumptions, there are no local pressure gradients and local filtration flows between volumes at $t = t_0$. Initial filtrate production (at $t = t_0$) occurs because of the liquid pressure gradient in the extraparticle volume. On the basis of Darcy's law, it is equal to the maximum value:

$$q_1(0,t_0) = -\frac{k_1}{\mu}\frac{\partial P_1(0,t_0)}{\partial z}. \tag{1}$$

As time increases, liquid pressures P_1, P_2, and P_3 decrease. Consequently, the effective pressure as applied on elementary particles, $P_{S1} = P_E - P_1$, increases. Obviously, this affects the displacement and reorientation of particles, and decreases the extraparticle porosity, ϵ_1. In reality, the equilibrium between the effective pressure P_{S1} and particle reorientation is obtained with some delay, leading to the assumption that consolidation occurs in two steps: a primary step and a secondary (or creep) step (Gibson and Lo, 1961; Shirato et al., 1974, 1978). The increase in P_{S1} also causes the compression of elementary particles and the decrease in both extracellular and intracellular porosities, ϵ_2 and ϵ_3, respectively.

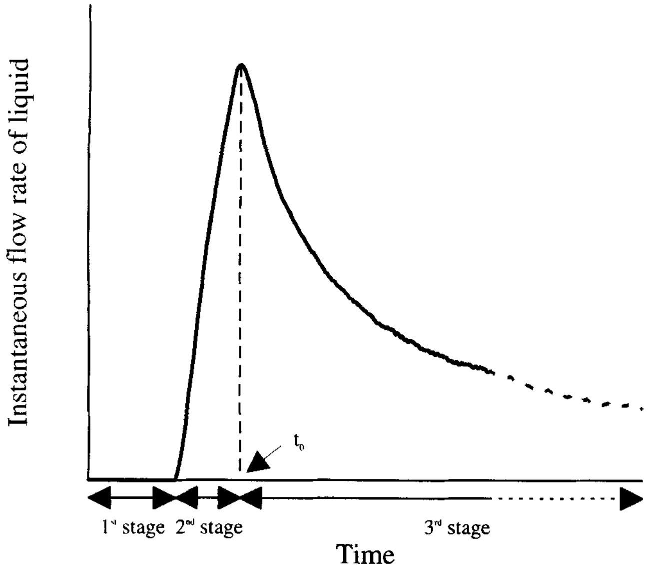

Figure 2. Instantaneous flow rate of liquid vs. time, in unidirectional (hydraulic) pressing.

Furthermore, the hydraulic conductivity of the extraparticle volume is much higher than that of the extracellular volume. And the hydraulic conductivity of the extracellular volume is much higher than that of the intracellular volume. Therefore, P_1 decreases faster than P_2, and P_2 decreases faster than P_3. Consequently, local pressure gradients exist, $(P_2 - P_1)/L$ and $(P_3 - P_2)/d$, which cause local filtration flows from the intracellular volume to the extracellular volume, and then from the extracellular volume to the extraparticle volume.

The additional effective stress, originating with the local pressure gradients between micro- and macropores tends to deform the porous blocks (Zheltov, 1966). In the case of cellular materials, the additional effective stress from the local pressure gradients tends to deform the intracellular and extracellular volumes.

Thus, the pressure difference $(P_2 - P_1)$ tends to expand the extracellular volume (increase of ϵ_2), and to compress the extracellular volume (decrease of ϵ_1); while the pressure difference $(P_3 - P_2)$ tends to increase ϵ_3 and decrease ϵ_2. Therefore,

$$\epsilon_1 = \epsilon_1(P_{S1}, \text{creep effects}, P_1 - P_2)$$
$$\epsilon_2 = \epsilon_2(P_{S1}, P_2 - P_1, P_2 - P_3)$$
$$\epsilon_3 = \epsilon_3(P_{S1}, P_3 - P_2).$$

The third stage of hydraulic pressing should be governed by four different steps:

- Primary consolidation of the extraparticle volume
- Creep consolidation of the extraparticle volume
- Deformation and deliquoring of the extracellular volume
- Deformation and deliquoring of the intracellular volume.

Mathematical Model

The model concerns the third stage of the hydraulic pressing, that is, the consolidation stage of the presscake.

Main assumptions

(a) Intracellular, extracellular, and extraparticle volumes are filled with liquid.

(b) Continuity equations for the liquid phase in the extraparticle, extracellular, and intracellular volumes are, respectively:

$$\frac{\partial(\rho\epsilon_1)}{\partial t} + \frac{\partial(\rho q_1)}{\partial z} - v_{2-1} = 0, \tag{2}$$

$$\frac{\partial(\rho\epsilon_2)}{\partial t} + \frac{\partial(\rho q_2)}{\partial z} + v_{2-1} - v_{3-2} = 0, \tag{3}$$

$$\frac{\partial(\rho\epsilon_3)}{\partial t} + \frac{\partial(\rho q_3)}{\partial z} + v_{3-2} = 0. \tag{4}$$

The form of the continuity equations, Eqs. 2–4, is typical for nonequilibrium mass-transfer processes in porous media, where υ_{2-1} or υ_{3-2} indicate the mass rate of substance per unit volume between interior and exterior pores (Farooq and Ruthven, 1991; Toupin et al., 1989; Barenblatt et al., 1990). In the case of solid/liquid expression, υ_{2-1} is a liquid mass rate per unit volume, which is a scalar product of two vectors: the vector of local liquid flow velocity from the extra cellular volume to the extra particle volume and the unit normal vector to a flow direction; υ_{3-2} is a scalar product of the vector of local liquid flow velocity from the intracellular volume to the extracellular volume and the unit normal vector to a flow direction.

At $\upsilon_{2-1}=0$ and $\upsilon_{3-2}=0$, continuity equations (Eqs. 2–4) transform to the classic Terzaghi form, which is widely used for filtration and expression processes (Terzaghi, 1925; Suklje, 1970; Shirato et al., 1974).

(c) Local liquid mass rates are defined by the following equations:

- From the extracellular to the extraparticle volume,

$$\upsilon_{2-1}=\psi_{2-1}(P_2-P_1). \tag{5}$$

- From the intracellular to the extracellular volume,

$$\upsilon_{3-2}=\psi_{3-2}(P_3-P_2). \tag{6}$$

Phenomenological equations (Eqs. 5 and 6) indicate that rates of liquid between intracellular, extracellular, and extraparticle volumes are proportional to the moving force of the process (pressure differences). Equations 5 and 6 correspond to the phenomenological equations of mass transfer between micro- and macropores (Farooq et al., 1991), through the cell wall (Toupin et al., 1989; Marcotte et al., 1991), and between porous blocks and rocks (Barenblatt et al., 1990).

(d) Local flow velocities of liquid in the extraparticle, extracellular, and intracellular volumes obey the Darcy filtration law:

$$q_1=-\frac{k_1}{\mu}\cdot\frac{\partial P_1}{\partial z}, \tag{7}$$

$$q_2=-\frac{k_2}{\mu}\cdot\frac{\partial P_2}{\partial z}, \tag{8}$$

$$q_3=-\frac{k_3}{\mu}\cdot\frac{\partial P_3}{\partial z}. \tag{9}$$

Equation 7 was used for the expression of cellular materials in numerous publications (Mrema et al., 1985; Schwartzberg, 1978, 1983; Rebouillat et al., 1993); it gives good agreement with experimental data for the primary consolidation period, when the filtrate production is greatest (Mrema et al., 1985; Lanoisellé et al., 1994). Therefore, the high-order counter parts of filtration law may be neglected.

(e) Extraparticle void ratio e_1 is a function of primary elastic stress (effective pressure P_{S1}), creep elastic and viscous stresses, and pressure difference (P_2-P_1). Then,

$$\frac{\partial e_1}{\partial t}=\left(\frac{\partial e_1}{\partial P_{S1}}\right)_{1_0}\frac{\partial P_{S1}}{\partial t}+\frac{\partial}{\partial t}\int_0^t\left(\frac{\partial e_1}{\partial P_{S1}}\right)_{1_1}\cdot\{1-e^{[-\nu_1(t-\tau)]}\}\cdot\frac{\partial P_{S1}}{\partial \tau}\,d\tau+\frac{\partial e_1}{\partial(P_1-P_2)}\cdot\frac{\partial(P_1-P_2)}{\partial t}. \tag{10}$$

The first term of the righthand side of Eq. 10 shows elastic primary consolidation of the extraparticle volume, which is only caused by the filtration resistance of expressed liquid. The classic Terzaghi theory and solid/liquid expression models for cellular materials (Mrema et al., 1985; Schwartzberg, 1978, 1979, 1983; Zaiats et al., 1993) are limited to this first member.

The second term of the righthand side of Eq. 10 indicates the retardation of the particle reorientation in the extraparticle volume due to the effect of creep (or secondary) consolidation. This effect is caused of the elastic and viscous resistance of a cake structure (Suklje, 1970), and depends on the history of the cake loading. The effect of creep consolidation was previously analyzed for the expression of mineral cakes (Shirato et al., 1974, 1978, 1986) and some cellular materials (Rebouillat, 1983). If the effective pressure increment at prior time τ is $(\partial P_{S1}/\partial\tau)d\tau$, then the void ratio increment at any time t due to creep consolidation is $(\partial P_{S1}/\partial\tau)\eta(z,\tau)d\tau$, where $\eta(z,\tau)$ is the function of creep. Function $\eta(z,\tau)$ increases with $(t-\tau)$ increasing. The usual form of this function is

$$\eta(t,\tau)=\left(\frac{\partial e_1}{\partial P_{S1}}\right)_1\cdot[1-e^{-\nu_1(t-\tau)}],$$

where $(\partial e_1/\partial P_{S1})_1$ and ν_1 (time factor) characterize, respectively, the elastic and the viscous properties of the cake structure (Suklje, 1970; Shirato et al., 1986).

The third term of the righthand side of Eq. 10 shows the deformation of the extraparticle volume due to pressure difference (P_1-P_2).

(f) Extracellular void ratio e_2 is a function of the effective pressure P_{S1} and liquid pressure differences (P_2-P_1) and P_2-P_3), so,

$$\frac{\partial e_2}{\partial t}=\frac{\partial e_2}{\partial P_{S1}}\cdot\frac{\partial P_{S1}}{\partial t}+\frac{\partial e_2}{\partial(P_2-P_1)}\cdot\frac{\partial(P_2-P_1)}{\partial t}+\frac{\partial e_2}{\partial(P_2-P_3)}\cdot\frac{\partial(P_2-P_3)}{\partial t}. \tag{11}$$

(g) Intracellular void ratio is a function of P_{S1} and liquid pressure difference (P_3-P_2), so,

$$\frac{\partial e_3}{\partial t}=\frac{\partial e_3}{\partial P_{S1}}\cdot\frac{\partial P_{S1}}{\partial t}+\frac{\partial e_3}{\partial(P_3-P_2)}\cdot\frac{\partial(P_3-P_2)}{\partial t}. \tag{12}$$

Methodology

On the basis of assumptions (a)–(g), the following set of equations can be developed for the expression of cellular materials:

$$\frac{1}{(1+e_1)^2G_{1_0}}\left(\frac{\partial P_1}{\partial t}-\frac{\partial P_E}{\partial t}\right)+\frac{\nu_1^2}{(1+e_1)^2G_{1_1}}\int_0^t\{[P_1(z,0)-P_E(0)]-[P_1(z,\tau)-P_E(\tau)]\}\cdot e^{-\nu_1(t-\tau)}d\tau$$

$$
-\frac{\nu_1}{(1+e_1)^2 G_{1_1}}\{[P_1(z,0)-P_E(0)]-(P_1-P_E)\}
$$

$$
+\frac{1}{(1+e_1)^2 E_1}\cdot\frac{\partial(P_1-P_2)}{\partial t}=\frac{\partial}{\partial z}\left(\frac{k_1}{\mu}\frac{\partial P_1}{\partial z}\right)
$$

$$
+\frac{\psi_2}{\rho}(P_2-P_1) \quad (13)
$$

$$
\frac{1}{(1+e_2)^2 G_2}\left(\frac{\partial P_1}{\partial t}-\frac{\partial P_E}{\partial t}\right)+\frac{1}{(1+e_2)^2 E_{2-1}}\cdot\frac{\partial(P_2-P_1)}{\partial t}
$$

$$
+\frac{1}{(1+e_2)^2 E_{2-3}}\cdot\frac{\partial(P_2-P_3)}{\partial t}=\frac{\partial}{\partial z}\left(\frac{\omega_2 k_1}{\mu}\frac{\partial P_z}{\partial z}\right)
$$

$$
-\frac{\psi_2}{\rho}(P_2-P_1)+\frac{\psi_3}{\rho}(P_3-P_2) \quad (14)
$$

$$
\frac{1}{(1+e_3)^2 G_3}\left(\frac{\partial P_1}{\partial t}-\frac{\partial P_E}{\partial t}\right)+\frac{1}{(1+e_3)^2 E_3}\cdot\frac{\partial(P_3-P_2)}{\partial t}
$$

$$
=\frac{\partial}{\partial z}\left(\frac{\omega_3 k_1}{\mu}\frac{\partial P_3}{\partial z}\right)-\frac{\psi_3}{\rho}(P_3-P_2), \quad (15)
$$

where

$$
G_{1_0}=-\left(\frac{\partial P_{S1}}{\partial e_1}\right)_{1_0}, \qquad G_{1_1}=-\left(\frac{\partial P_{S1}}{\partial e_1}\right)_{1_1},
$$

$$
G_2=-\left(\frac{\partial P_{S1}}{\partial e_2}\right), \qquad G_3=-\left(\frac{\partial P_{S1}}{\partial e_3}\right),
$$

$$
E_1=\frac{\partial(P_1-P_2)}{\partial e_1}, \qquad E_{2-1}=\frac{\partial(P_2-P_1)}{\partial e_2},
$$

$$
E_{2-3}=\frac{\partial(P_2-P_3)}{\partial e_2}, \qquad E_3=\frac{\partial(P_3-P_2)}{\partial e_3}.
$$

Equation 13 characterizes a piezoconductivity of the extraparticle volume. The first term in the lefthand part of this equation shows the influence of elastic primary consolidation of the extraparticle volume (G_{1_0}) on its piezoconductivity. The second and third terms, which result from the integration by parts of the integral of Eq. 10, show the influence of creep consolidation (G_{1_1}, ν_1) on the piezoconductivity of the extraparticle volume. The last term on the left side of Eq. 13 shows the influence of expansibility of the extraparticle volume (E_1) on its piezoconductivity. The terms in the righthand side of Eq. 13 show the influence of permeability of the extraparticle volume (k_1) and intensity of liquid flow from the extracellular volume to the extraparticle volume (ψ_2) on the piezoconductivity of the extraparticle volume.

Equation 14 characterizes a piezoconductivity of the extracellular volume. Members in the left side of this equation show the influence of elastic consolidation (G_2) and expansibilities (E_{2-1}, E_{2-3}) of the extracellular volume on its piezoconductivity. Terms on the right side of Eq. 14 show the influence of the permeability of the extracellular volume ($k_2 = \omega_2 k_1$) and intensity of liquid flow from the intracellular volume to the extracellular volume (ψ_3) and from the extracellular volume to the extraparticle volume (ψ_2) on the piezoconductivity of the extracellular volume.

Equation 15 characterizes a piezoconductivity of the intracellular volume. Terms on the left side of this equation show the influence of elastic consolidation (G_3) and expansibility (E_3) of the intracellular volume on its piezoconductivity. Terms on the right side of Eq. 15 show the influence of permeability of the intracellular volume ($k_3 = \omega_3 k_1$) and intensity of liquid flow from the intracellular volume to the extracellular volume (ψ_3) on the piezoconductivity of the intracellular volume.

Cell volume and extracellular channels represent the main resistance in expression of cellular materials (Mrema et al., 1985). Therefore, the permeability of the extraparticle volume (k_1) is considerably more than the permeabilities of the extracellular (k_2) and intracellular (k_3) volumes. In this case parameters ω_2 and ω_3 in Eqs. 14 and 15 may be considered minor parameters ($\omega_2 = (k_2/k_1) \ll 1$, $\omega_3 = (k_3/k_1) \ll 1$). Consequently, the liquid flow velocities at the scale of intracellular and extracellular volumes may be neglected; then $q_2 = q_1 = 0$.

Assuming that the compressibility and expansibility moduli and local resistances are constant, we can represent the solution of Eq. 15 as follows:

$$
P_3-P_2=\alpha_3\int_0^t\frac{\partial(P_E-P_1)}{\partial\tau}\cdot e^{-\nu_3(t-\tau)}d\tau \qquad (16)
$$

where $\alpha_3 = E_3/G_3$ and $\nu_3 = \psi_3 E_3(1+e_3)^2$.

Equation $t_{R_3} = 1/\nu_3$ is the retardation time for the deliquoring of intracellular volume. Integrating Eq. 16 by parts gives

$$
P_3(z,t)=P_2(z,t)+\alpha_3\Big\{[P_E(t)-P_1(z,t)]-[P_E(0)-P_1(z,0)]
$$

$$
\cdot e^{-\nu_3 t}-\nu_3\int_0^t[P_E(\tau)-P_1(z,\tau)]\cdot e^{-\nu_3(t-\tau)}d\tau\Big\}. \quad (17)
$$

Substituting Eq. 17 into Eq. 13, and after some rearrangements, one can obtain

$$
\frac{\partial(P_2-P_1)}{\partial t}+\nu_2(P_2-P_1)+\nu_3(\beta_2+\beta_3)(P_3-P_2)
$$

$$
=\alpha_2\frac{\partial(P_E-P_1)}{\partial t}
$$

where

$$
\alpha_2=\frac{E_{2-1}}{G_2}, \qquad \beta_2=\frac{E_{2-1}}{E_{2-3}}, \qquad \beta_3=\frac{E_{2-1}}{E_3}\frac{(1+e_2)^2}{(1+e_3)^2},
$$

$$
\nu_2=\psi_2 E_{2-1}(1+e_2)^2. \quad (18)
$$

Equation $t_{R_2} = 1/\nu_2$ represents the retardation time for deliquoring of extracellular volume. Assuming that $t_{R_3} \gg t_{R_2}$, then $\nu_2 \gg \nu_3$. In this case, the third term of the left side of Eq. 18 may be neglected compared to the second member. The solution of the simplified form of Eq. 18 becomes

$$P_2 - P_1 = \alpha_2 \int_0^t \frac{\partial (P_E - P_1)}{\partial \tau} \cdot e^{-\nu_2 (t-\tau)} d\tau. \tag{19}$$

Integrating Eq. 19 by sides gives

$$P_2(z,t) = P_1(z,t) + \alpha_2 \Big\{ [P_E(t) - P_1(z,t)] - [P_E(0) - P_1(z,0)] \cdot e^{-\nu_2 t} - \nu_2 \int_0^t [P_E(\tau) - P_1(z,\tau)] \cdot e^{-\nu_2 (t-\tau)} d\tau \Big\}. \tag{20}$$

Then, substituting Eq. 20 into Eq. 13, we obtain

$$\frac{\partial (P_1 - P_E)}{\partial t} + \delta_1 \nu_1^2 \int_0^t \{ [P_1(z,0) - P_E(0)] - [P_1(z,\tau) - P_E(\tau)] \} \cdot e^{-\nu_1 (t-\tau)} d\tau - \delta_1 \nu_1 \{ [P_1(z,0) - P_E(0)] - (P_1 - P_E) \} + \nu_2 \left(\frac{1}{\alpha_1} + \frac{1}{\alpha_{2-1}} \right) (P_2 - P_1) = b_{1_0} \frac{\partial^2 P_1}{\partial z^2},$$

where

$$\delta_1 = \frac{G_{1_0}}{G_{1_1}}, \qquad \alpha_1 = \frac{E_1}{G_{10}}, \qquad \alpha_{2-1} = \frac{E_{2-1}}{G_{1_0}} \frac{(1+e_2)^2}{(1+e_1)^2},$$

$$b_{1_0} = \frac{k_1}{\mu} G_{1_0} (1+e_1)^2. \tag{21}$$

Equation $t_{R_1} = 1/\nu_1$ represents the retaration time of the creep consolidation. Assuming that $t_{R_2} \gg t_{R_1}$, then $\nu_1 \gg \nu_2$. In this case, the last term of the left side of Eq. 21 can be neglected.

After differentiating the simplified Eq. 21 with respect to t, we obtain

$$\frac{\partial^2 (P_1 - P_E)}{\partial t^2} + \delta_1 \nu_1^3 \int_0^t \{ [P_1(z,0) - P_E(0)] - [P_1(z,\tau) - P_E(\tau)] \} \cdot e^{-\nu_1 (t-\tau)} d\tau - \delta_1 \nu_1^2 \{ [P_1(z,0) - P_E(0)] - (P_1 - P_E) \} + \delta_1 \nu_1 \frac{\partial (P_1 - P_E)}{\partial t} = b_{1_0} \frac{\partial^3 P_1}{\partial z^2 \partial t}. \tag{22}$$

Dividing the simplified Eq. 21 (where $\nu_2 = 0$) by ν_1, and adding the result to Eq. 22, gives

$$\frac{1}{\nu_1} \left(\frac{\partial^2 P_1}{\partial t^2} - \frac{\partial^2 P_E}{\partial t^2} \right) + (1+\delta_1) \left(\frac{\partial P_1}{\partial t} - \frac{\partial P_E}{\partial t} \right) = \frac{b_{1_0}}{\nu_1} \frac{\partial^3 P_1}{\partial z^3 \partial t} + b_{1_0} \frac{\partial^2 P_1}{\partial z^2}. \tag{23}$$

It should be noted that Eq. 23 is similar to the basic equation of the creep consolidation theory (Florin, 1959; Suklje, 1970) at P_E constant. Also, Eq. 23 is similar to the basic consolidation equation of Terzaghi (1925), at $t_{R_1} = 1/\nu_1 = 0$.

Boundary conditions for Eq. 23 are

$$\frac{\partial P_1}{\partial z}(h,t) = 0. \tag{24}$$

No flow from the cake to the hydraulic piston

$$P_1(0,t) = 0. \tag{25}$$

Pressure P_E is transferred to the particles at the interface between the cake and the filter medium ($P_E = P_{S1}$ and $P_1 = P_E - P_{S1} = 0$). The first initial condition consists of setting $t = 0$ in Eq. 21:

$$\frac{\partial P_1(z,0)}{\partial t} = b_{1_0} \frac{\partial^2 P_1(z,0)}{\partial z^2}. \tag{26}$$

Recall that $\nu_2 = 0$ in Eq. 21.

The creep consolidation theory in soil mechanics (Florin, 1959; Suklje, 1970) and the expression theory from homogeneous semisolid materials (Shirato et al., 1986) assume that initially the applied pressure P_E is transferred only to the liquid phase ($P_E = P_1(z,t_0)$; $P_{S1}(z, t_0) = 0$). But this assumption does not explain the expression of liquid from cellular materials during the first and second stages of the process. The initial liquid as expressed from the intracellular and extracellular volumes can take place just if $P_{S1}(z, t_0) \neq 0$. Therefore, the second initial condition can be postulated (Lanoisellé et al., 1994):

$$P_1(z,t_0) = [P_E(0) - P_{S0}] \cdot \frac{z}{h}. \tag{27}$$

The solution of Eq. 23 for the constant-pressure expression (P_E = const), with the boundary and initial conditions of Eqs. 24–27 then has the form

$$P_1(z,t) = (P_E - P_{S0}) \sum_{i=1}^{\infty} \frac{8(-1)^{i+1}}{\pi^2 (2i-1)^2} (R_i'' \cdot e^{\gamma_i' t} - R_i' \cdot e^{\gamma_i'' t}) \cdot \sin \lambda_i z, \tag{28}$$

where

$$R_i' = \frac{\gamma_i' + b_{1_0} \lambda_i^2}{\gamma_i'' - \gamma_i'},$$

$$\gamma_i' = -M_i + \sqrt{M_i^2 - N_i},$$

$$M_i = \frac{1}{2} \left[\nu_1 (1+\delta_1) + b_{1_0} \lambda_i^2 \right],$$

$$\lambda_i = \frac{\pi (2i-1)}{2h}, \qquad i = 1,2,\dots.$$

$$R_i'' = \frac{\gamma_i'' + b_{1_0} \lambda_i^2}{\gamma_i'' - \gamma_i'},$$

$$\gamma_i'' = -M_i - \sqrt{M_i^2 - N_i},$$

$$N_i = \nu_i b_{1_0} \lambda_i^2, \tag{29}$$

Substituting Eq. 28 into Eq. 20, gives

$$P_2(z,t)=(P_E-P_{S0})\sum_{i=1}^{\infty}\frac{8(-1)^{i+1}}{\pi^2(2i-1)^2}\left\{\alpha_2\cdot e^{-\nu_2 t}+(1-\alpha_2)\right.$$
$$\times[R_i''\cdot e^{\gamma_i' t}-R_i'\cdot e^{\gamma_i'' t}]+\frac{\alpha_2\nu_2R_i''}{\gamma_i'+\nu_2}(e^{\gamma_i' t}-e^{-\nu_2 t})$$
$$\left.-\frac{\alpha_2\nu_2R_i'}{\gamma_i''+\nu_2}(e^{\gamma_i'' t}-e^{-\nu_2 t})\right\}\sin(\lambda_i z). \qquad (30)$$

Substituting Eqs. 28 and 30 into Eq. 15, gives

$$P_3(z,t)=(P_E-P_{S0})\sum_{i=1}^{\infty}\frac{8(-1)^{i+1}}{\pi^2(2i-1)^2}\left\{(\alpha_2\cdot e^{-\nu_2 t}+\alpha_3\cdot e^{-\nu_3 t})\right.$$
$$+(1-\alpha_2-\alpha_3)[R_i''\cdot e^{\gamma_i' t}-R_i'\cdot e^{\gamma_i'' t}]$$
$$+\frac{\alpha_2\nu_2R_i''}{\gamma_i'+\nu_2}(e^{\gamma_i' t}-e^{-\nu_2 t})-\frac{\alpha_2\nu_2R_i'}{\gamma_i''+\nu_2}(e^{\gamma_i'' t}-e^{-\nu_2 t})$$
$$\left.+\frac{\alpha_3\nu_3R_i''}{\gamma_i'+\nu_3}(e^{\gamma_i' t}-e^{-\nu_3 t})-\frac{\alpha_3\nu_3R_i'}{\gamma_i''+\nu_3}(e^{\gamma_i'' t}-e^{-\nu_3 t})\right\}$$
$$\times\sin(\lambda_i z). \qquad (31)$$

Deformation and consolidation of the presscake

The total deformation of the presscake occurs due to the deformation of the extraparticle, extracellular, and intracellular volumes. The deformations of these volumes can be determined separately by Eqs. 10, 11, and 12, respectively.

- For the extraparticle volume:

$$S_1(t)=\int_0^h\int_{e_1}^{e_1(z,0)}de_1\cdot dz=\frac{1}{G_{1_0}}\int_0^h\int_{P_1}^{P_1(z,0)}dP_1\cdot dz$$
$$-\frac{\nu_1}{G_{1_1}}\int_0^h\int_J^0 dJ\cdot dz+\frac{1}{E_1}\int_0^h\int_{P_1-P_2}^0 d(P_1-P_2)\,dz, \qquad (32)$$

where

$$J=\int_0^t[P_1(z,0)-P_1(z,\tau)]\cdot e^{-\nu_1(t-\tau)}d\tau.$$

- For the extracellular volume:

$$S_2(t)=\int_0^h\int_{e_2}^{e_2(z,0)}de_2\cdot dz=\frac{1}{G_2}\int_0^h\int_{P_1}^{P_1(z,0)}dP_1$$
$$\cdot dz+\frac{1}{E_{2-1}}\int_0^h\int_{P_2-P_1}^0 d(P_2-P_1)\,dz$$
$$+\frac{1}{E_{2-3}}\int_0^h\int_{P_2-P_3}^0 d(P_2-P_3)\,dz. \qquad (33)$$

- For the intracellular volume:

$$S_3(t)=\int_0^h\int_{e_3}^{e_3(z,0)}de_3\cdot dz=\frac{1}{G_3}\int_0^h\int_{P_1}^{P_1(z,0)}dP_1\cdot dz$$
$$+\frac{1}{E_3}\int_0^h\int_{P_3-P_2}^0 d(P_3-P_2)dz. \qquad (34)$$

The total deformation of the presscake during hydraulic pressing is given by:

$$S(t)=S_1(t)+S_2(t)+S_3(t). \qquad (35)$$

Substituting the values of $P_1(z,\ t)$, $P_2(z,\ t)$, $P_3(z,\ t)$ from Eqs. 27–29 into Eqs. 32–34 allows us to obtain the final total layer deformation:

$$S_\infty=S(\infty)=(1+\delta_1+\delta_2+\delta_3)\cdot\frac{P_E-P_{S0}}{2G_{1_0}}\cdot h \qquad (36)$$

and the total layer consolidation for a constant-pressure expression:

$$U=\frac{V(t)}{V_\infty}=\frac{S(t)}{S_\infty}$$
$$=\frac{1}{1+\delta_1+\delta_2+\delta_3}\left\{1-\sum_{i=1}^{\infty}\frac{32(-1)^{i+1}}{\pi^3(2i-1)^3}[R_i''\cdot e^{\gamma_i' t}-R_i'\cdot e^{\gamma_i'' t}]\right\}$$
$$+\frac{\sum_{k=1}^{3}\delta_k(1-e^{-\nu_k t})}{1+\delta_1+\delta_2+\delta_3}$$
$$-\frac{1}{1+\delta_1+\delta_2+\delta_3}\sum_{k=1}^{3}\sum_{i=1}^{\infty}\delta_k\nu_k\frac{32(-1)^{i+1}}{\pi^3(2i-1)^3}$$
$$\times\left[\frac{R_i''}{\gamma_i'+\nu_k}(e^{\gamma_i' t}-e^{-\nu_k t})-\frac{R_i'}{\gamma_i''+\nu_k}(e^{\gamma_i'' t}-e^{-\nu_k t})\right], \qquad (37)$$

where

$$\delta_2=\frac{G_{1_0}}{G_2}, \qquad \delta_3=\frac{G_{1_0}}{G_3}.$$

Equation 37 can be simplified in assuming that the retardation time of the creep consolidation ($t_{R_1}=1/\nu_1$) is considerably greater than the retardation time of the primary consolidation of the extraparticle volume [$t_{R_0}=(4h^2/\pi^2b_{1_0})$]. In this case, $\nu_1\ll(\pi^2b_{1_0}/4h^2)$ and the coefficients of Eq. 29 become $R_i'=-1$; $R_i''=0$; $\gamma_i'=0$; $\gamma_i''=-b_{1_0}\lambda_i^2$. The simplified form of Eq. 37 for the total layer consolidation can then be written:

$$U=\frac{1}{1+\delta_1+\delta_2+\delta_3}\left[1-\sum_{i=1}^{\infty}\frac{32(-1)^{i+1}}{\pi^3(2i-1)^3}\cdot e^{-b_{1_0}\lambda_i^2 t}\right]$$

$$+\frac{\delta_1}{1+\delta_1+\delta_2+\delta_3}(1-e^{-\nu_1 t})+\frac{\delta_2}{1+\delta_1+\delta_2+\delta_3}(1-e^{-\nu_2 t})$$

$$+\frac{\delta_3}{1+\delta_1+\delta_2+\delta_3}(1-e^{-\nu_3 t}). \quad (38)$$

The first term on the righthand side of Eq. 38 shows the primary consolidation of the extraparticle volume, while the second term represents the creep consolidation of the extraparticle volume, the third term the consolidation of the extracellular volume, and the last term the consolidation of the intracellular volume.

Analogy with linear viscoelastic models

Equation 38 can be rearranged as follows:

$$U=\left[\frac{1}{\frac{1}{G_{1_0}}+\frac{1}{G_{1_1}}+\frac{1}{G_2}+\frac{1}{G_3}}\right]$$

$$\times\left[\frac{1}{G_{1_0}}\left(1-\sum_{i=1}^{\infty}\frac{32(-1)^{i+1}}{\pi^3(2i-1)^3}\cdot e^{-b_{1_0}\lambda_i^2 t}\right)\right.$$

$$\left.+\frac{1}{G_{1_1}}(1-e^{-\nu_1 t})+\frac{1}{G_2}(1-e^{-\nu_2 t})+\frac{1}{G_3}(1-e^{-\nu_3 t})\right]. \quad (39)$$

The series of the first number on the righthand side converges very rapidly, and then Eq. 39 gives

$$U=\left[\frac{1}{\frac{1}{G_{1_0}}+\frac{1}{G_1}+\frac{1}{G_2}+\frac{1}{G_3}}\right]\left[\sum_{i=1_0,1_1,2,3}\frac{1}{G_i}(1-e^{-\nu_i t})\right] \quad (40)$$

where $\nu_{1_0}=b_{1_0}(\pi^2/4h^2)$.

Equation 40 is equivalent to the generalized Kelvin model, with four Kelvin elements in series, as developed for describing the creep behavior of linear viscoelastic materials, under constant compressive stress. The Kelvin elements physically represent the four consolidation steps: the primary and creep consolidation of the extraparticle volume, and the consolidations of both extracellular and intracellular volumes. Each element is characterized by physical constants G_i, the reciprocal compressibility modulus or compliance, and $t_{R_i}=1/\nu_i$, the retardation time. Both constants represent the presscake properties. Their identification allows us discussion of the presscake's behavior, according to the cellular structure of the raw materials.

Case Study: Hydraulic Pressing of Rapeseed

The aforementioned model will be used to investigate the characteristics of the hydraulic pressing of rapeseed. The experimental apparatus and methodology were described earlier in detail (Lanoisellé et al., 1994) and are summarized in the following paragraphs.

The hydraulic press (Figure 3) is build around a chamber (1) where 200 g of seeds can be placed. An internal 140-mm-diameter jack (2) generates pressure (up to 150 bars), which is measured with a sensor (3) (Kulite-JPB, type C610RA, Bois d'Arcy, France) placed in the hydraulic oil inlet (4). Both piston (5) and chamber are thermoregulated (30° to 130°C ± 2°C) using thermal oil heating with the boiler (Parmilleux type H10, Lyon, France). Expressed oil is continuously measured with a centigram precision balance (Mettler-Toledo type PM6000, Viroflay, France). Output signals (jack pressure, chamber pressure, expressed oil mass) are recorded and computerized at a frequency of one hertz.

Presently, the Liquid-Containing Biporous Particles Expression Model will be applied for an expression experiment carried out at $P_E = 75$ bar and $T = 95°C$, during a period of 406 minutes. Figure 4 shows the resulting expression curves, which are pressure vs. time and instantaneous flow rate of liquid (derived from oil mass) vs. time. Due to the high frequency of data acquisition and the precision of the balance, this flow rate needed to be numerically smooth (using a 20-point moving average) to determine its maximum. Then the three stages of the expression process are clearly recognized. It is also obvious that the applied pressure, P_E, is rather constant during the third, or consolidation, stage.

Identification of consolidation characteristics assuming that $\nu_3 \ll \nu_2 \ll \nu_1 \ll b_{1_0}(\pi^2/4h^2)$ allows the simplification of Eq. 39; at large expression times, it gives

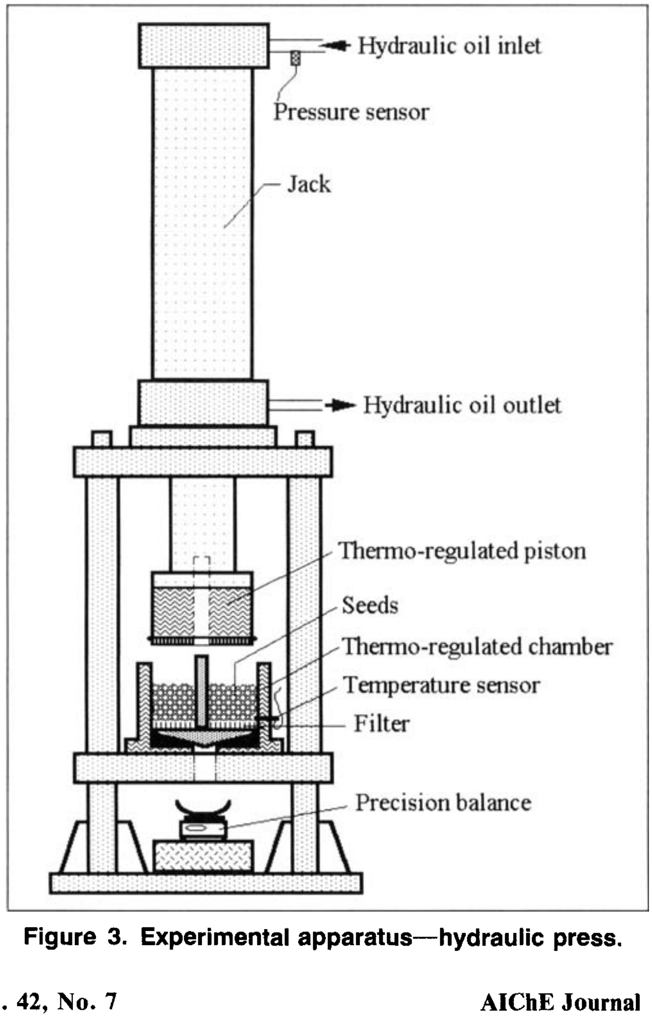

Figure 3. Experimental apparatus—hydraulic press.

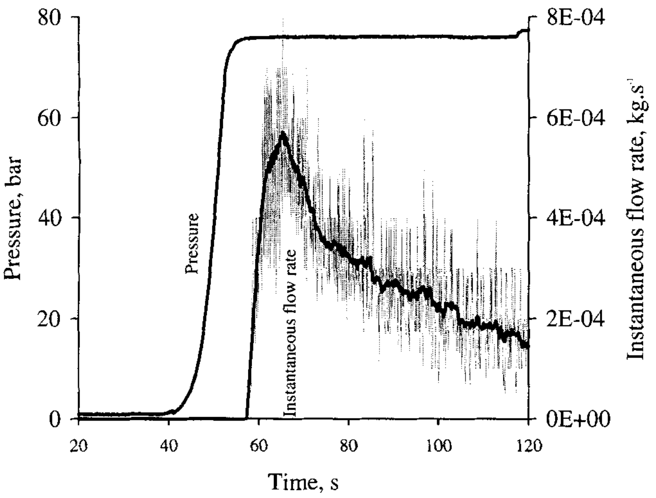

Figure 4. Experimental expression curves for rapeseed.

$P_E = 75$ bar; $T = 95°C$.

$$U = \frac{V}{V_\infty} = 1 - C\cdot e^{-\nu_3 t}. \quad (41)$$

Let t_0 be equal to zero at the beginning of the consolidation stage, and V_∞ estimated by interpolation as defined by Lanoisellé et al. (1994). Figure 5a presents the experimental pressing consolidation curve for rapeseed that is the evolution of $\ln(1-U)$ vs. time. At relatively small expression times, the curve shows an important, nonlinear decrease with time. At relatively large expression times, from about 11,000 s (t_3) up to 24,000 s, the curve is perfectly linear, as predicted by Eq. 41. This means that, at high expression times, the presscake consolidation is governed by the deformation and deliquoring of the intracellular volume.

The characteristics of the linear part of the curve allow determination of the retardation time, $t_{R_3} = (1/\nu_3)$ (from the slope), and the coefficient C (from the ordinate at $t_0 = 0$), which relates to the compressibility modulus, G_3.

Analyzing the experimental consolidation curve at lower expression times allows the identification of the physical characteristics of the first, second, and third steps. In fact, when subtracting the fourth term of the righthand side of Eq. 39 to U, the consolidation ratio becomes

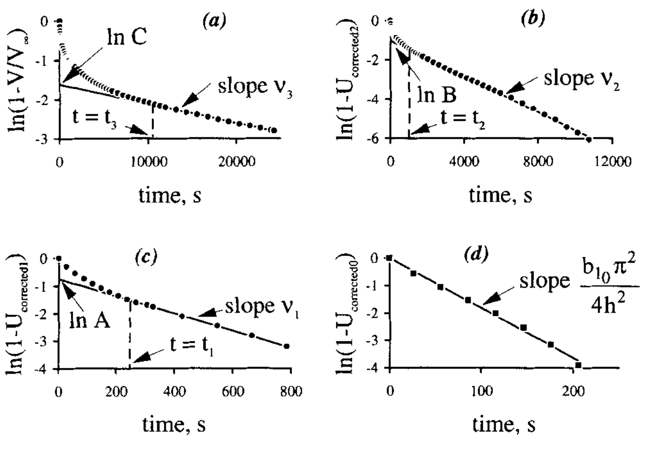

Figure 5. Expression consolidation curves for rapeseed.

$P_E = 75$ bar; $T = 95°C$. (a), (b), (c), (d): ■, experiment; —, Eq. 39.

$$U_{\text{corrected2}} = \frac{U - C(1-e^{-\nu_3 t})}{1-C}$$

$$= (1-B)\left\{(1-A)\left[1-\sum_{i=1}^{\infty}\frac{32(-1)^{i+1}}{\pi^3(2i-1)^3}\cdot e^{-b_{1_0}(\pi^2(2i-1)^2/4h^2)t}\right] + A(1-e^{-\nu_1 t})\right\} + B(1-e^{-\nu_2 t}) \quad (42)$$

The corrected consolidation ratio, $U_{\text{corrected2}}$, represents the presscake consolidation for the consolidation of extraparticle volume, and the deformation and deliquoring of extracellular volume.

Figure 5b, which shows $\ln(1-U_{\text{corrected2}})$ vs. time, behaves linearly at an expression time higher than $t \cong 1{,}200$ s, as predicted by Eq. 43:

$$U_{\text{corrected2}} = 1 - B\cdot e^{-\nu_2 t}. \quad (43)$$

Then, the retardation time $t_{R_2} = 1/\nu_2$ and the coefficient B can be determined from the linear part of experimental consolidation curve shown in Figure 5b. From t_2 up to t_3, the expression of presscake is governed by the deformation and deliquoring of the extracellular volume.

The same procedure is repeated for estimating the retardation time t_{R_1}, the coefficient A (from $\ln(1-U_{\text{corrected1}})$ vs. time, Figure 5c), and the retardation time t_{R_0}, the primary consolidation coefficient b_{1_0} (from $\ln(1-U_{\text{corrected0}})$ vs. time, Figure 5d):

$$U_{\text{corrected1}} = \frac{U_{\text{corrected2}} - B(1-e^{-\nu_2 t})}{1-B}$$

$$= (1-A)\left[1-\sum_{i=1}^{\infty}\frac{32(-1)^{i+1}}{\pi^3(2i-1)^3}\cdot e^{-b_{1_0}(\pi^2(2i-1)^2/4h^2)t}\right] + A(1-e^{-\nu_1 t}), \quad (44)$$

where the corrected consolidation ratio, $U_{\text{corrected1}}$, represents the two steps of the consolidation of extraparticle volume. From t_1 ($\cong 240$ s) up to about 1,200 s, the presscake consolidation is mainly governed by the creep consolidation of extraparticle volume, as predicted by

$$U_{\text{corrected1}} = 1 - A\cdot e^{-\nu_1 t}. \quad (45)$$

Finally,

$$U_{\text{corrected0}} = \frac{U_{\text{corrected1}} - A(1-e^{-\nu_1 t})}{1-A}$$

$$= 1-\sum_{i=1}^{\infty}\frac{32(-1)^{i+1}}{\pi^3(2i-1)^3}\cdot e^{-b_{1_0}(\pi^2(2i-1)^2/4h^2)t}, \quad (46)$$

where the corrected consolidation ratio, $U_{\text{corrected}0}$, represents the presscake consolidation at low expression times. From 0 up to about 240 s, the presscake consolidation is governed by the primary consolidation of the extraparticle volume, as predicted by Eq. 47:

$$U_{\text{corrected}0} \cong 1 - e^{-b_{1_0}(\pi^2/4h^2)}, \qquad (47)$$

with

$$\nu_0 = b_{1_0}\frac{\pi^2}{4h^2}.$$

This identification procedure allows us to determine the basic physical characteristics of the consolidation stage, the retardation times, and compressibility moduli in particular, as presented in Table 1. It is noted that the primary consolidation of the extraparticle volume is almost instantaneous. As an analogy with the rheology of viscoelastic solid materials, the primary consolidation would relate to some elasticity of the presscake, releasing very easily and rapidly some amount of free extractable oil. Then, the presscake would behave like a three-element Kelvin solid, as the time of mechanical expression increases. Each element would correspond to the extraparticle, extracellular, and intracellular volumes, and present very different characteristics, or retardation times, from low retardation time at the particle scale to high retardation time at the cell scale. In fact, the retardation time represents the characteristic time of oil flow in the cellular network. The resistance to liquid flow is much higher within the cells than in the interstitial volume.

The intracellular volume shows the higher compressibility modulus, as it deforms during expression, while the extracellular volume reveals the lower compressibility modulus. In fact, the intracellular volume relates to the cell membrane skeleton of the cellular material. This is the more rigid and compact part of the material. The extraparticle volume shows an intermediate compressibility modulus as it refers to the deformation of the whole particles.

Liquid pressure in the presscake

Figure 6 shows the evolution of liquid pressures vs. square time for the extraparticle, extracellular, and intracellular volumes, as calculated at $z = h$, through Eqs. 28, 30, and 31, respectively. As can be seen, liquid pressure decreases rapidly in the extraparticle volume, more slowly in the extracellular volume, and very slowly in the intracellular volume. This is in good agreement with the retardation spectrum of the presscake, as identified earlier.

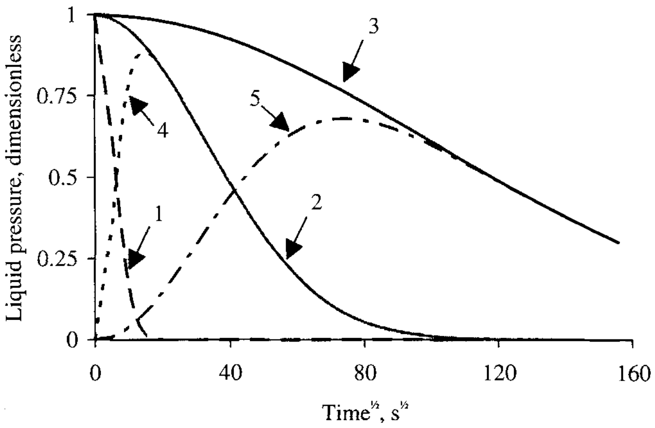

Figure 6. Dimensionless liquid pressure as a function of (time)$^{1/2}$.

P_E = 75 bar; T = 95°C; rapeseed.
Curve 1: liquid pressure with the extraparticle volume; Curve 2: liquid pressure with the extracellar volume; Curve 3: liquid pressure with the intracellular volume; Curve 4: pressure drop between extracellular and extraparticle volumes; Curve 5: pressure drop between intracellular and extracellular volumes.

Figure 6 also shows the pressure drops between the different volumes of the presscake, between extracellular and extraparticle volumes (curve 4), and between intracellular and extracellular volumes (curve 5). Both pressure-drop curves present a maximum that means that in the consolidation stage the flow rate of liquid between the related volumes increases up, to a maximum value and then decreases, as the expression time increases.

Conclusion

The mechanical expression of cellular materials is quite complicated due to a cell network that makes the drainage of the liquid to be extracted complex. Consequently, the classic models, as developed for mineral-based materials, no longer describe the expression of cellular materials. This justifies the development of a specific model, the Liquid-Containing Biporous Particles Expression Model. This model describes the effect of the cellular structure on liquid flow, accounting for three different volumes: the extraparticle, extracellular, and intracellular volumes. This leads to modeling the presscake consolidation, which is very important as it determines the expression performances.

As shown in Figure 7, the developed model is in good agreement with the experimental data (at constant pressure) for short as well as for long expression times. Figure 7 also demonstrates the pertinence of the four-step consolidation

Table 1. Physical Characteristics of the Consolidation Stage

Consolidation Steps	Retardation Time min	Compressibility Modulus bar
Primary consolidation, extraparticle vol.	0.92	414.4
Creep consolidation, extraparticle vol.	5.29	453.2
Deformation and deliquoring, extracellular vol.	36.55	386.6
Deformation and deliquoring, intracellular vol.	336.1	556.5

P_E = 75 bar; T = 95°C; t = 406 min.

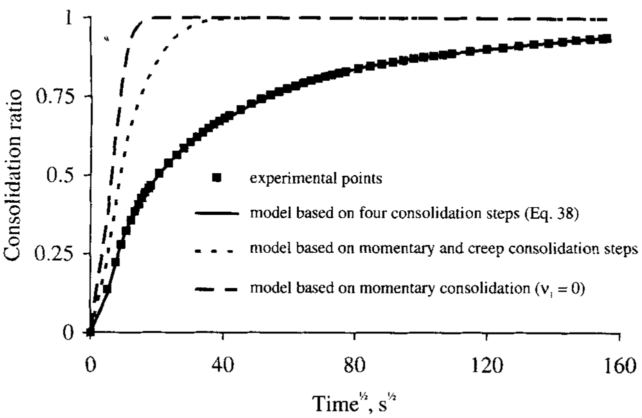

Figure 7. Models vs. experimental data.

P_E = 75 bar; T = 95°C; rapeseed.

model, as compared with other simplified models that were developed earlier with one or two consolidation steps (Lanoisellé et al., 1994).

It is impossible to predict the real value of V_∞ for cellular materials, based on one- or two-steps models (Lanoisellé et al., 1994). On the other hand, substitution of the real value of V_∞ into the conventional consolidation equations shows poor agreement with experimental data (Figure 7). One can favorably use the conventional two-steps Terzaghi–Voight model (Shirato et al., 1986) if the experiment is terminated after the second consolidation step and $t_{R_1} \ll t_{R_2} \ll t_{R_3}$. In this case, the value of V_∞, predicted on the basis of the two-step model, should be substituted into the conventional consolidation equation. In the same way, one can favorably use the classic Terzaghi model (Mrema et al., 1985) if experiment is terminated after the first consolidation step and $t_{R_0} \ll t_{R_1} \ll t_{R_2} \ll t_{R_3}$. In this case, the value of V_∞, predicted on the basis of the one-step model, should be substituted into the Terzaghi consolidation equation.

The four-step model may be used to describe the behavior of any biological and agrofood raw materials when submitted to mechanical expression. Then it would allow prediction of the expressibility of the cellular liquid-containing materials, and optimize the expression performances as well.

Notation

d = characteristic length of cell, m
M_0 = initial mass of liquid, kg
P_S = local solid pressure, Pa
P_{S0} = initial local solid pressure, at $z = h$, Pa
q = local liquid velocity, m·s^{-1}
T = temperature, °C
V = volume of liquid expressed at time t, m^3
V_∞ = theoretical maximal volume of liquid expressed, m^3
z = linear coordinate, m

Greek letters

α_1, α_{2-1}, α_2, α_3 = ratios of expansibility and compressibility moduli (Eqs. 20, 18, 16)
β_2, β_3 = ratios of expansibility moduli (Eq. 18)
δ_1, δ_2, δ_3 = ratios of compressibility moduli (Eqs. 21, 36)
φ = shape factor
μ = liquid viscosity, Pa·s
ρ = liquid density, kg·m^{-3}
σ = total mechanical stress, Pa
σ_c = contacted stress, Pa

Subscripts and Superscripts

1_0 = values of primary consolidation parameters in extraparticle volume
1_1 = values of creep consolidation parameters in extraparticle volume
$'$ = modified parameters

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